Article

A Study Of Bed Occupancy Management In The Healthcare System Using The M/M/C Queue And Probability

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Abstract

Many diverse healthcare systems, including hospitals, laboratories, pharmaceutical business, radiology, organ transplant etc. have been modelled and studied using queuing models. Due to their analytical nature and capacity for speedy resolution, queueing models are well-liked by researchers and system designers and can offer reasonably accurate assessments of system performance. The problem of allocating the ideal number of beds to different wards, as well as assigning roles to doctors in various hospital wards and outside the hospital, is one that health care management is now dealing with. In this paper, we will use (M/M/C) queue to explore bed occupancy management, patient movement, and ideal bed counts in hospitals. Additionally, we will look for the probability that a patient will be chosen for a bed allocation and the expected occupancy rates for a specific time frame.

Keywords

M/M/C Queue, Bed Occupancy, Patients Movement, Probability, Queuing Theory, Expectation

Introduction

The queueing hypothesis is crucial to the management of healthcare. The health care system can be compared to a network of queues with various types of servers where patients arrive in a finite or infinite manner, wait for services (such as treatment, routine checkups, etc.), and then receive a result, such as going home or being admitted to a hospital, after consulting with a doctor. In any medical clinic, there are typically two ways that people arrive: either by appointment or on their own. Those who need only

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medical advice, outdoors treatments, time to time checkup by doctors they may wait without causing excessive worry either to them or to the hospital management. However, those patients who need emergency service like accident, maternity, Intensive care unit, etc. don't come with prior information, and they need to admit in hospital and long waiting for them may cause loss of one's life.

Theoretical Model

Firstly, we will discuss the problem of bed occupancy, movement of patients in a hospital for finding optimal bed count in a hospital with an acceptable delay.

Here we consider M/M/C queue or (Erlang –C model) [1] . Let m be the Fixed no. of beds in hospital, λ be the mean arrival rate of patients, μ be the mean service time of patients. Here service represents allotment of beds to needed patients, arrival and service distribution of patients follow Poisson's distribution. Let average length of stay per patient in hospital =T. Then, Average no. of arrival of patients during an average length of stay T is $a = \frac{\lambda}{T}$,which is referred as Offered load [10]. No queuing is allowed as the number of beds are fixed. If a patient finding all c beds occupied on arrival, then he/she will be considered as lost.

Probability that out of m beds, r beds are occupied is given by $P_r = \frac{\frac{a^r}{r!}}{\sum_{k=0}^{m} \frac{a^k}{k!}}$,where r=0,1,2,……..m and $P_r = 0$, for r>m……..[2] [3][11]

And the Probability that all m beds are occupied is $P_m = \frac{\frac{a^m}{m!}}{\sum_{k=0}^{m} \frac{a^k}{k!}}$

Also, by Erlang's loss formula, the probability that all m beds are occupied i.e. The proportion of arrival of patients which is lost from the hospital is $B(m,a) = \frac{\frac{a^m}{m!}}{\sum_{k=0}^{m} \frac{a^k}{k!}}$ ………[4][10] and

The mean no. of occupied beds is a'=a[1-B (m,a)]…………[5][10]which is also referred as carried load.

Since, the carried load (CL) is equal to the proportion of the offered load which is not lost from the system.[10]

∴If the no. of beds is infinite i.e. m=∞. Then, carried load=offered load i.e. a=a′

Also, Lost load=aB(m,a)……[6][10]

Hence, the proportion of arrival of patients which is lost from the hospital is $B(m,a) = \frac{aB(m,a)}{a} = \frac{\text{Lost load}}{\text{offered load}}$

Since, system is in steady state, therefore, $\rho \le 1$, & we define bed occupancy in hospital as $\rho = \frac{a'}{c}$ …….[7][10]

Table 1. Shows Different Characteristics for Available Beds

No. of available Beds(m)	Bed occupancy (ρ)	Lost probability B(m,a)
10	3.06E-10	1
20	0.1	1.53E-09
30	0.066667	1.43E-20
40	0.05	4.78E-33
50	0.04	1.31E-46

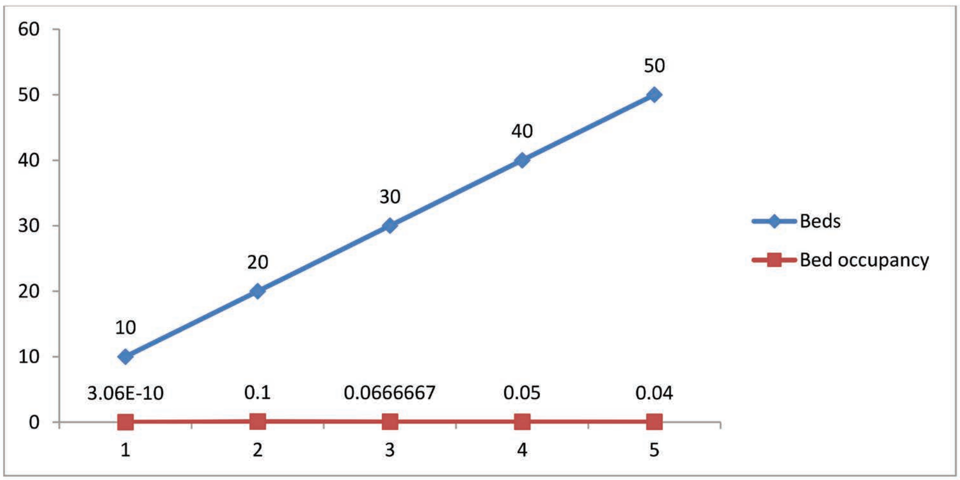

Fig. 1(a). Bed occupancy vs available beds

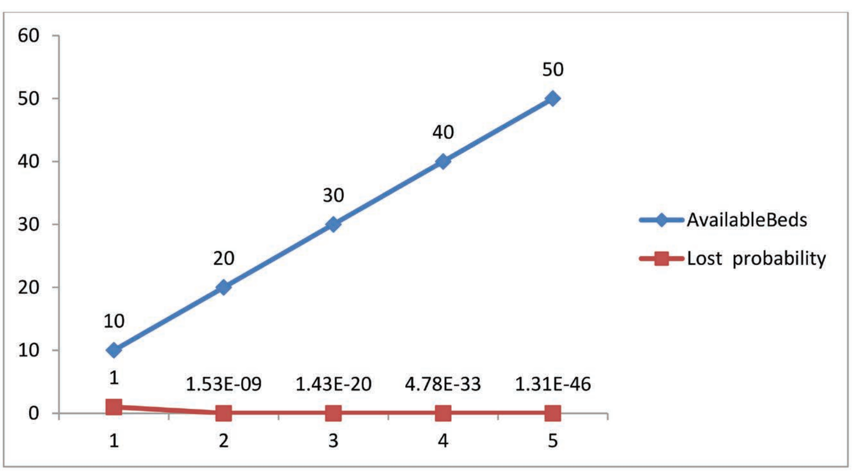

Fig. 1 (b). Lost probability vs available beds

Application of Above Model:

Let arrival rate is λ=30 patients / day and let average stay length is T=10 days in a hospital.

Then a=offered load=3

Now, we use probability theory to determine expected number of beds on each day in a hospital for a particular time period.

Here we are doing a calculation only for a week.

Let $S=\{X_1, X_2, \ldots\ldots\ldots X_{100}\}$ be the sample space. Here probability of each bed allotment is same and it is equal to $P(x_i)=\frac{1}{10}$, i=1,2,3,………..,100.

Expected number of occupied beds in each week is

E(occupied beds)= $\sum_{i=1}^{100} iP(x_i)=\frac{1}{101}(1+2+3+\ldots\ldots\ldots\ldots 1$
00)=50 i.e. 50 beds are occupied in a week.

Let N=100 be the available number of beds to the hospital.

Then On 1st day of a week, the probability of a single bed allotment $=\frac{1}{N+1}=\frac{1}{101}$ and

Expected number of occupied beds is 50 out of 100 in a week.

So, we can assume on an average, 7 beds are allotted on day 1 of a week.

So, on 2nd day of a week,93 beds remain unoccupied.

Therefore, the probability of a single bed allotment $=\frac{1}{N+1}=\frac{1}{94}$ and

Expected number of occupied beds for remaining 6 Days= $\sum_{i=1}^{93} iP(x_i)=\frac{1}{94=}(1+2+3+\ldots\ldots\ldots\ldots 93)=46$ app.

On 3rd day of a week,86 beds remain unoccupied,

$\therefore$The probability of a single bed allotment $=\frac{1}{N+1}=\frac{1}{87}$ and

Expectednumberofoccupiedbedsforremaining5Days= $\sum_{i=1}^{86} iP(x_i)=\frac{1}{87}(1+2+3+\ldots\ldots\ldots\ldots 86)=43.$

On 4th day of a week,79 beds remain unoccupied and

$\therefore$The probability of a single bed allotment $=\frac{1}{N+1}=\frac{1}{80}$ and

Expectednumberofoccupiedbedsforremaining4Days = $\sum_{i=1}^{79} iP(x_i)=\frac{1}{80}(1+2+3+\ldots\ldots\ldots\ldots 79)=39.$ app.

On 5th day of a week,72 beds remain unoccupied and

$\therefore$ The probability of a single bed allotment $=\frac{1}{N+1}=\frac{1}{73}$ and

Expected number of occupied beds for remaining 3 Days

$$= \sum_{i=1}^{72} iP(x_1) = \frac{1}{73}(1+2+3+...........72) = 36 \text{ . app.}$$

On 6th day of a week, 65 beds remain unoccupied and

$\therefore$The probability of a single bed allotment $= \dfrac{1}{N+1} = \dfrac{1}{66}$ and

Expected number of occupied beds for remaining 2 Days

$$= \sum_{i=1}^{65} iP(x_i) = \frac{1}{66}(1+2+3+...........65) = 32. \text{ app.}$$

On 7th day of a week, 58 beds remain unoccupied and

$\therefore$The probability of a single bed allotment $= \dfrac{1}{N+1} = \dfrac{1}{59}$ and

Expected number of occupied beds for remaining 1 Day

$$= \sum_{i=1}^{58} iP(x_i) = \frac{1}{59}(1+2+3+...........58) = 29. \text{ app.}$$

Result: From table 1, fig.1(a), 1(b) it is clear, the bed occupancy and the proportion of arrived patients that are lost decreases with an increase in number of beds. If we further increases number of beds then B(m,a) eventually reaches to zero.

From table 2, fig (2), it is clear as the probability of a bed allotment on each day of a week increases, the expected number of beds on each day of a week increases.

Conclusion

Here, we draw the conclusion that as the number of beds increases, bed occupancy declines, along with the percentage of patients who arrive but are turned away or lost. Additionally, as the probability of a bed allotment on each day of a week increases, the expected number of beds on each day as well increases This study concludes that our model may be beneficial for solving bed occupancy problem in hospitals. As a result, more patients can be admitted to the hospital and receive the necessary medical care.

Table 2. Probability and Expected no. of beds for each day of a week (when N=Total no. of beds=100)

Week Days	P(x)	Expected no. of beds
Day 1	0.0099	7
Day 2	0.0106	8
Day 3	0.0114	9
Day 4	0.0125	10
Day 5	0.0136	12
Day 6	0.01515	16
Day 7	0.01694	29

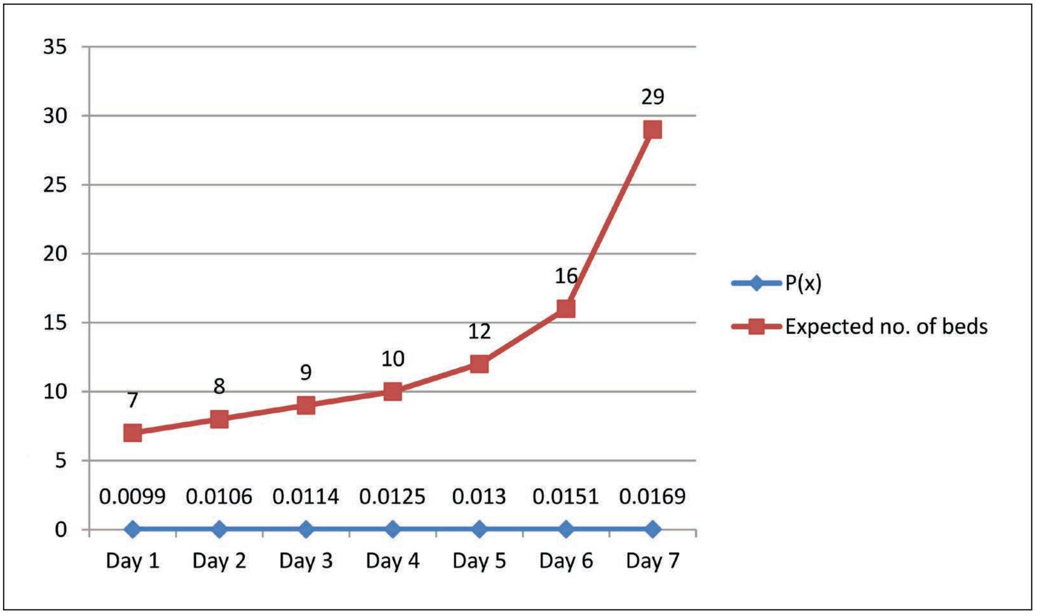

Fig. 2. Expected no. of bed counts with probability for a week

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