Article

Analysis of Single Server Markovian Queueing Model with Differentiated Working Vacation, Vacation Interruption, Soft Failure, Reneging of Customers

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Abstract

This article investigates the reneging of customers in M/M/1 model with differentiated working vacation, vacation interruption and soft failure. The customers come with rate λ and receives service during busy period with rate μ, where λ and μ obeys markovian distribution. In this model two distinct vacations are considered: one has been taken just after serving all customers in busy period with slow service rate θ as some soft failure occurs during working vacation (Vacation I). At an epoch of completion of working vacation, if any customers are present in the system, then the server moves to busy period for serving customers otherwise move to vacation II. During vacation II if customer comes then interruption is assumed to occur in the vacation and server returns to busy period otherwise remain in vacation .When an arriving customer finds server is on working vacation, it makes customer impatient and it start up an impatient timer T_0 with an exponentially distributed rate α_0 If service does not begin before T_0 expires, the customer might renege with probability p without getting served or wait for their turn with probability 1-p=q. By using PGF technique we have derived different steady state probabilities and various system performances analytically. Effect of few parameters on different system performances have been shown numerically and illustrated graphically.

Keywords

Differentiated Vacation, Working Vacation, Vacation Interruption, Reneging, Soft Failure.

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1. Introduction

Queueing systems featuring server vacation provided a broad platform for recent advances in research in queueing theory. Server vacation in queueing theory plays a significant role in our everyday lives. Due to the intense rivalry that exists in the service industry these days, the primary goal of companies that offers services across all categories is to give clients better service in a shorter amount of time. There are several circumstances in queueing system where instead of completely stopping service the server continues to operate during vacation at some slower rate. This type of vacation is termed as working vacation. Recently, Ibe et al. [21] proposed notion of differentiated vacations for first in which two different kinds of vacations are taken by server: one is taken right after a busy period and second is taken when there remain no customers in queue during first vacation. They considered first vacation longer as comparison to second vacation. Later, the authors Ibe et al. [21] introduced the notion of vacation interruption in the model of Isijola et al. [13]. Some fundamental structures of differentiated vacation have shown in the research work of Fiems et al. [6] and Vishenvsky et al. [31]. Now a days, vacation interruption has become very important in using the server as efficiently as possible. Queueing systems with different variant of vacation interruption have been investigated by Baba [37], Chen et al. [10], Zhang et al. [11]. In queueing systems, reneging behavior refers to the condition where customers leave the queue before receiving the service. This behavior of customers can occur due to various reasons, including long waiting time, perceived service quality, unforeseen circumstances, competing priorities, service abandonment etc. Haight [12] first have done work on reneged customers in a one server markovian queue. Researchers can refer to Robert [26], Abou-El-Ata et al. [3] for additional information and research on this topic.

2. Brief Literature Review

A brief review of work done by researchers in past few years are discussed here:

Levy et al. [36] were the first who introduced the notion of vacations in the queueing systems. Working vacation was proposed first by Servi et al. [17]. One could refer research papers of Baba [35], Wu et al. [7], Banik et al. [2], Liu et al [34] for knowing tremendous work done in area of working vacation in few past years. Vijayashree et al. [14] presented transient solution of a markovian queueing system having single server with differentiated vacation by using second kind of modified Bessel's function. Further, Vijayashree et al. [15] included the concept of interruption in differentiated vacation in a M/M/1 queueing system with the concept of threshold policy. Unni et al. [32,33] studied differentiated vacation in queueing system with more than one server and interruption during differentiated working vacation when customer reaches to some prespecified value respectively. Aissani et al. [1] used Laplace transform and generating function to obtain queue size and steady state probabilities of server states in a queueing system with differentiated vacation. Sampath et al. [28] discussed impatience in customers during differentiated vacation. Li et al. [19] introduced the concept of working vacation interruption for an $M/M/1$ queue. Zhang and Hou [20] dealt with interruption during working vacations in a $M/G/1$ queue. Li et al. [18] analyzed the $GI/M/1$ queue with working vacations and vacation interruption. Ayyappan et al. [4] used notion of vacation interruption in an $M/M/1$ retrial queue with k-phase Erlang distribution. Sreenivasan et al. [29] initially proposed the notion of thresholds in one server markovian queue, in which vacation interruption occur when the size of queue reaches a specified value. Manoharan et al. [22] discussed the concept of setup time and interruption during Bernoulli scheduled working vacation. Poonam et al. [23] analyzed balking and working vacation interruption in bernoulli schedule in a single server retrial queueing model.

Kumar et al. [16] studied retention of reneging customers in 'c' server finite length queue. Mishra et al. [27] used confluent hypergeometric function, continued fraction, generating function method to study the balking and reneging behavior probabilistically in a M/M/1 system. Srivastava et al. [30] extended work of Mishra et al. [27] by using various special function, Laplace transform and generating method under multiple differentiated vacations in a M/M/1 system. Faud et al. [9] discussed the reneging behavior of customers and their retention during a working vacation in a M/M/1/∞ model. Swathi et al. [5] analyzed reneging behavior of customer with single and multiple vacation policies in a single server markovian queueing system. Kumar et al. [24] studied reneging and retention of discouraged arrivals in a single server finite capacity markovian queue. Further Kumar et al. [25] extended his work with balking in a multiple server queuing system. Yang et al. [8] used matrix method and Fourth order Runge Kutta's method in study of retention of reneged customers when server goes to breakdown and repair during servicing.

3.1 Model Description

Consider a M/M/1 Queueing system with differentiated working vacation, soft failure, vacation interruption and reneging of customer. Customers arrive according to a Poisson process with rate λ, Service time during busy periods follows an exponential distribution with rate μ. Queue discipline is first come first serve. customers are served in the order they arrive. In our model we are considering two types of server vacation: First one is working vacation which is taken immediately after a busy period in which due to soft failure server provides service at slow rate θ, where θ is exponentially distributed. The server goes for a single working vacation when server becomes free after serving all customers, where vacation time is exponentially distributed with parameter ξ.At an instant of working vacation completion, if customers are present in the system then the server goes back to busy period for regular service otherwise go to vacation II. During vacation II if customer comes then vacation is assumed to be interrupted and server resumes service in busy period otherwise remain in vacation II with exponentially distributed rate γ. Customers who arrive during working vacation becomes impatient as they experience slower service and activates an impatient timer T_0 which starts ticking with an exponentially distributed rate α_0.If service does not begin before T_0 expires,the customer may renege with probabilituy p without getting served or wait in the system for their turn with probability q=1-p.The rate Transition diagram of the model is shown in Fig.1.

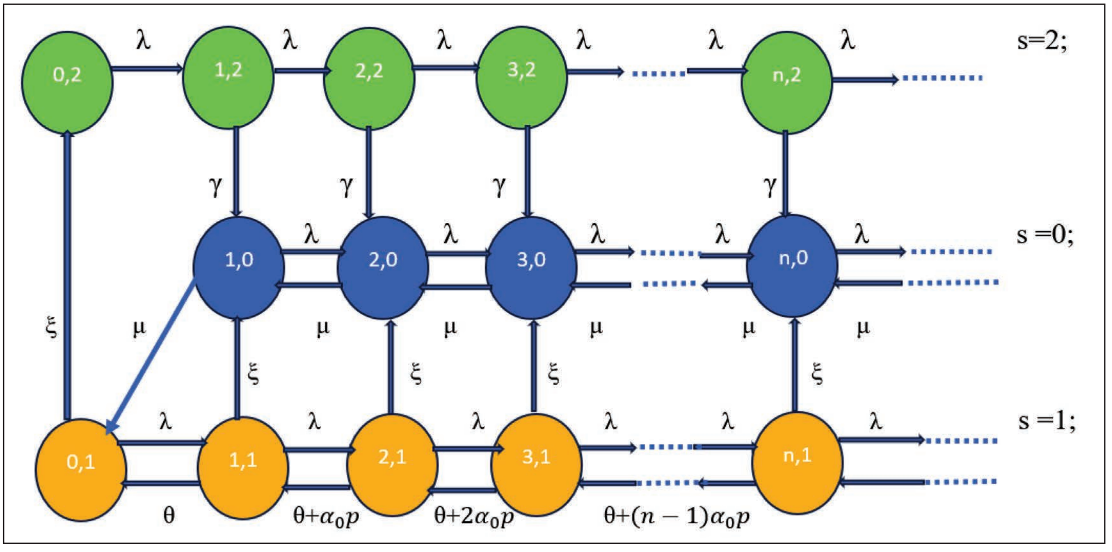


Figure 1: The rate transition diagram of the model with different server states.

3.2 Mathematical Formulation

Let P_{ns} = P{$N(t) = n, S(t) = s; n = 0,1,2 \ldots \ldots \ldots; s = 0,1,2$} denotes the steady or equilibrium state probabilities, where N(t) represents customers number at time t, S(t) represents state of server at time t in the structure(system)such that

s=0 denotes busy period state.

s=1 denotes working vacation state.

s=2 denotes vacation II state.

Balance equations for each server state are given as follows:

For s=0;

$$(\lambda+\mu)P_{10} = \gamma P_{12} + \xi P_{11} + \mu P_{20}, n = 1 \tag{1}$$

$$(\lambda+\mu)P_{n0} = \gamma P_{n2} + \xi Pn_{1} + \mu P_{n+10} + \lambda P_{n-10}, n \geq 2 \tag{2}$$

For s = 1;

$$(\lambda+\xi)P_{01} = \mu P_{10} + \theta P_{11}, n = 0 \tag{3}$$

$$\{\lambda+\xi+\theta+(n-1)\alpha_0 p\} P_{n1} = \lambda P_{n-11} + (\theta + n\alpha_0 p)P_{n+11}, n \geq 1 \tag{4}$$

For s=2;

$$\lambda P_{02} = \xi P_{01}, n = 0 \tag{5}$$

$$(\lambda+\gamma)P_{n+12} = \lambda P_{n2}, n \geq 1 \tag{6}$$

3.3 Steady State Probabilities and Some System Performances

Now we use Probability generation technique to derive various system performances and steady state probabilities,

$$\text{Let } K_0(z) = \sum_{n=1}^{\infty} P_{n0} z^n, K_0'(z) = \sum_{n=1}^{\infty} nP_{n0} z^{n-1} \tag{7}$$

$$K_1(z) = \sum_{n=0}^{\infty} P_{n1} z^n, K_1'(z) = \sum_{n=0}^{\infty} nP_{n1} z^{n-1} \tag{8}$$

$$K_2(z) = \sum_{n=0}^{\infty} P_{n2} z^n, K_2'(z) = \sum_{n=0}^{\infty} nP_{n2} z^{n-1} \tag{9}$$

$$\text{such that } K_0(1) + K_1(1) + K_2(1) = 1. \tag{10}$$

Multiply equation (4) by z^n and taking summation over n, we get,

$$\sum_{n=1}^{\infty}\left[\lambda+\xi+\theta+(n-1)\alpha_0 p\right]P_{n1}z^n.=\lambda\sum_{n=1}^{\infty}P_{n-11}z^n.+\sum_{n=1}^{\infty}\left(\theta+n\alpha_0 p\right)P_{n+11}z^n$$

On further simplifying above equation we get,

$$K_1(z)=\frac{[(\lambda+\xi+\theta-\alpha_0 p)z+\alpha_0 p-\theta]P_{01}-\theta zP_{11}}{[(\lambda+\xi+\theta-\alpha_0 p-\lambda z)z+\alpha_0 p-\theta]} \tag{11}$$

Taking $\lim_{z\to 1}K_1(z)$ in (11) we get,

$$K_1(1)=\frac{(\lambda+\xi)P_{01}-\theta P_{11}}{\xi}=P(\mathrm{WV}) \tag{12}$$

where P(WV)=$K_1(1)$=Probability of server is in period of working vacation.

Now, differentiate equation (11) once and taking limit z→1,

$$K_1'(1)=\frac{1}{\xi}[(\lambda+\xi+\theta-p\alpha_0)P_{01}-\theta P_{11}-K_1(1)(\xi+\theta-\alpha_0 p-\lambda)] \tag{13}$$

where $K'(1)=E(L_1)$= Expected length of queue in Working vacation.

Now, differentiate equation (11) twice and put limit z → 1

$$K_1''(1)=\frac{2}{\xi}\left[\lambda K_1(1)-K_1'(1)(\xi+\theta+\lambda-p\alpha_0)\right] \tag{14}$$

Multiply equation (6) by z^n and taking summation over n, we get,

$$\sum\nolimits_{n=1}^{\infty}\lambda P_{n2}z^n=\sum\nolimits_{n=1}^{\infty}(\lambda+\gamma)P_{n+12}z^n$$

On further simplifying above equation we get,

$$K_2(z)=\frac{(\lambda+\gamma)\left\{zP_{12}+P_{02}\right\}-\lambda zP_{02}}{\left[\lambda(1-z)+\gamma\right]} \tag{15}$$

Taking $\lim_{z\to 1}K_2(z)$ in (11) we get,

$$K_2(1)=\frac{(\lambda+\gamma)P_{12}+\gamma P_{02}}{\gamma}$$
$$=\left(\frac{\lambda+\gamma}{\lambda\gamma}\right)\xi P_{01}=\mathrm{P(V)} \tag{16}$$

where $K_2(1)$ =P(V)= Probability of server is on vacation II.

Now, differentiate equation (15) once and put limit z→ 1,

$$K_2'(1) = \left(\frac{\lambda+\gamma}{\gamma}\right)^2 P_{12} \tag{17}$$

where $K_2'(1) = E(L_2) =$ Expected length of queue in vacation II.

Now, differentiate equation (15) twice and put limit z→ 1,

$$K_2''(1) = \frac{1}{\gamma^4}\left[P_{12}(3\lambda^2\gamma^2 + \lambda\gamma^3 + \lambda^2 + \lambda\gamma + 2\lambda^3\gamma) + P_{02}(\gamma^2\lambda^2 - \lambda^2)\right] \tag{18}$$

Multiply equation (2) by z^n and taking summation over n,we get

$$(\lambda+\mu)\sum_{n=2}^{\infty} P_{n0} z^n = \gamma\sum_{n=2}^{\infty} P_{n2} z^n + \xi\sum_{n=2}^{\infty} P_{n1} z^n + \mu\sum_{n=1}^{\infty} P_{n+10} z^n + \lambda \sum_{n=2}^{\infty} P_{n-10} z^n$$

On further simplifying above equation we get,

$$(z-1)(\mu-\lambda z)K_0(\mathrm{z}) - (\lambda+\mu)z^2 P_{10} = z[\frac{-\xi\gamma P_{01}}{\lambda} - \frac{\xi\gamma z P_{01}}{\lambda+\gamma} + \gamma K_2(\mathrm{z}) - \xi P_{01} - \xi z P_{11} + \xi K_1(z) - \mu P_{10}] \tag{19}$$

Taking limit $z \to 1$ in equation (18) we get

$$K_0(1) = \left(\frac{\xi}{\mu-\lambda}\right) P_{01} = \mathrm{P(B)} \tag{20}$$

$K_0(1)$ =P(B)= Probability of server being in busy period.

Now, differentiate equation (19) both side w.r.t. z and put limit $z \to 1$ in it, we get

$$K_0'(1) = \frac{1}{2(\mu-\lambda)}[2\lambda K_0(1) + 2(\lambda+\mu)P_{10} - 2\frac{\xi\gamma}{\lambda+\gamma}P_{01} + \gamma K_2''(1) + 2\gamma K_2'(1) - 2\xi P_{11} + \xi K_1''(1) + 2\xi K_1'(1) = \mathrm{E}(L_0)$$

where E (L_0)= Expected length of queue when server is in busy period.

The expected length of queue in the system is E(L)= E (L_0)+ E (L_1)+ E (L_2).

Mean Sojourn time of system is $\mathrm{W} = \frac{\mathrm{E(L)}}{\lambda}$

The average reneging rate R of customers during period of working vacation is given by

$$\mathrm{R} = \sum_{n=0}^{\infty}\left[(n-1)\alpha_0 p\right]P_{n1}$$
$$= \alpha_0 p\left[K_1'(1) - K_1(1)\right]$$

The proportion of lost customer during period of working vacation is given by

$$\mathrm{P(L)} = \frac{R}{\lambda}$$

From recurrence relation (1), (2), (3), (4), (5), (6), we get,

$$P_{02} = \left(\frac{\xi}{\lambda}\right)P_{01} = F_1 P_{01}$$

$$P_{12} = \left(\frac{\xi}{\lambda+\gamma}\right)P_{01} = F_2 P_{01}$$

$$P_{10} = \left(\frac{\xi}{\mu}\right)P_{01} = F_3 P_{01}$$

$$P_{11} = \frac{1}{\theta}\left[\lambda - \gamma K_1(1) + \gamma K_2(1) + \xi\right]P_{01} = F_4 P_{01}$$

By using above *Pnj's*, we can rewrite $K_0(1)$, $K_1(1)$, $K_2(1)$ and P_{11} in terms of P_{01}, as follows,

$$K_2(1) = \left(\frac{\lambda+\gamma}{\lambda\gamma}\right)\xi P_{01} = B_2 P_{01}, \text{ where } B_2 = \left(\frac{\lambda+\gamma}{\lambda\gamma}\right)\xi$$

$$K_0(1) = \left(\frac{\xi}{\mu-\lambda}\right)P_{01} = B_0 P_{01}, \text{ where } B_0 = \left(\frac{\xi}{\mu-\lambda}\right)$$

$$K_1(1) = \left(\frac{(\lambda+\xi)P_{01} - \theta F_4 P_{01}}{\zeta}\right) = B_1 P_{01}, \text{ where } B_1 = \left(\frac{(\lambda+\xi)-\theta F_4}{\xi}\right)$$

∵ $K_0(1)$, $K_1(1)$, $K_2(1)$, $K'_0(1)$, $K'_1(1)$, $K'_2(1)$, P_{10}, P_{11}, P_{02}, P_{12} all are expressed in terms of P_{01}, therefore we need to calculate P_{01} which can be determined by normalization condition,

$$K_0(1) + K_1(1) + K_2(1) = 1$$

$$B_0 P_{01} + B_1 P_{01} + B_2 P_{01} = 1,$$

$$[B_0 + B_1 + B_2]\, P_{01} = 1$$

$$\therefore P_{01} = [B_0 + B_1 + B_2]^{-1},$$

$$\text{where, } B_0 = \left(\frac{\xi}{\mu-\lambda}\right), B_1 = \left(\frac{(\lambda+\xi)-\theta F_4}{\xi}\right), B_2 = \left(\frac{\lambda+\gamma}{\lambda\gamma}\right)\xi.$$

4. Numerical Analysis

Table 1: Impact of second vacation rate γ on system probabilities P(B), P(WV), P(V).

γ	P(B)	P(WV)	P(V)
0.05	0.01	0.000999	0.406666
0.1	0.01	0.001994	0.206656
0.25	0.01	0.004999	0.086662
0.5	0.01	0.01	0.046664
1	0.01	0.02	0.02666

Table 1 shows effect of second vacation rate γ on system probabilities P(B), P(WV), P(V) for ξ=1. It is clear that, when we take ξ=1 and increase values of γ for λ=3, θ=0.2, μ=5 α_0=0.4, p=0.3, P(B) remains constant, P(WV) changes with very slow increment while decrement have been observed in P(V).

Table 2: Impact of second vacation rate γ on $E(L_0)$, $E(L_1)$, $E(L_2)$, E(L), W, R, P(L) for ξ=1.

γ	$E(L_0)$	$E(L_1)$	$E(L_2)$	E(L)	W	R	P(L)
0.05	175.998	0.038719	3721	3897.036	1299.01	0.004526	0.0015086
0.1	56.1051	0.039839	906.01	962.1549	320.71	0.00454	0.001513
0.25	24.2487	0.043199	169	193.2919	64.43	0.00458	0.001526
0.5	13.9187	0.0488	49	62.9675	20.98	0.004656	0.001552
1	9.63698	0.06	16	25.69698	8.56	0.0048	0.0016

Table 2 shows effect of second vacation rate γ on various system performances $E(L_0)$, $E(L_1)$, $E(L_2)$, E(L),W, R, P(L) for ξ=1. It is clear that when we take λ=3, θ=0.2, μ=5, α_0=0.4,p=0.3 and fix ξ=1 then as γ increases $E(L_0)$,$E(L_2)$,E(L),W decreases while $E(L_1)$ increases and a very small(negligible) increment have been observed in value of R and P(L).

Table 3: Impact of reneging probability p on P(B), P(WV), P(V), R, P(L) for γ=ξ=1.

p	P(B)	P(WV)	P(V)	R	P(L)
0.1	0.01	0.033334	0.04	0.002111	0.000703
0.3	0.01	0.033334	0.04	0.006207	0.002069
0.5	0.01	0.033334	0.04	0.010133	0.003377
0.7	0.01	0.033334	0.04	0.013886	0.004628
0.9	0.01	0.033334	0.04	0.017469	0.005823

Table 3 shows effect of reneging probability p on P(B),P(WV),P(V),R,P(L) for γ=ξ=1.It is clear that when we take λ=3,θ=0.2, μ=5, α_0=0.4, as p increases, P(B),P(WV),P(V) remains constant while a very small increment have been observed in R and P(L).

5. Graphical Illustration

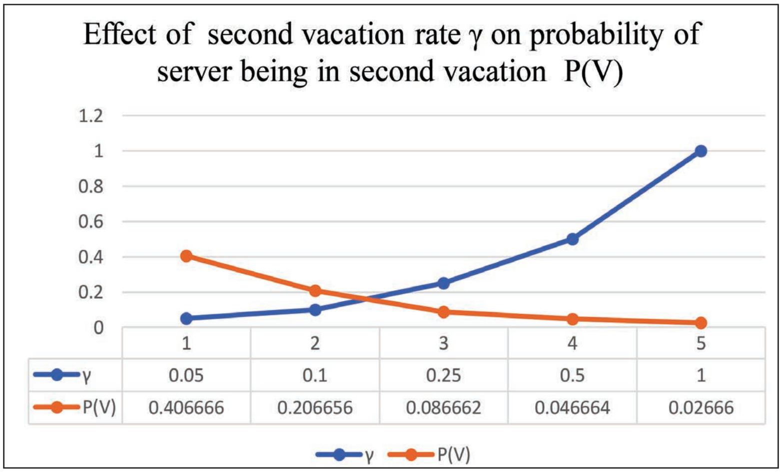


Figure 2: Impact of second vacation rate γ on probability of server being in second vacation P(V).

Fig. 2 shows impact of second vacation rate γ on probability of server being busy during second vacation P(V) for ξ=1, where γ≤ξ. It has been observed from Fig. 2 that when we take λ=3, θ=0.2, μ=5, α_0=0.4, p=0.3 and fix ξ=1 (where γ≤ξ) as γ increases, P(V) i.e. Probability of server being busy during second vacation decreases.

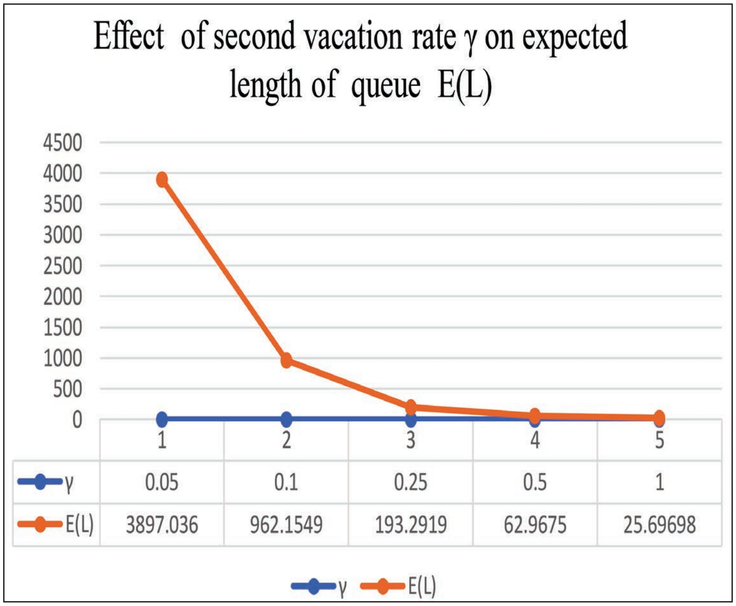
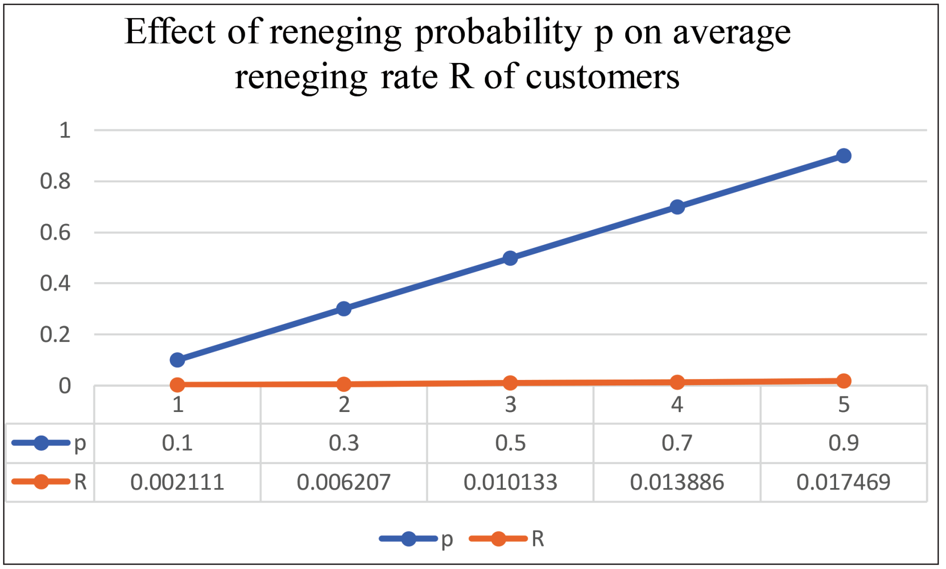


Figure 3: Impact of second vacation rate γ on expected length of queue E(L).

Fig. 3 shows impact of second vacation rate γ on expected length of queue E(L) for ξ=1 where γ≤ξ. It has been observed from Fig. 3 that when we take λ=3, θ=0.2, μ=5, α_0 =0.4, p=0.3 and fix ξ=1 (where γ≤ξ) as γ increases, E(L) i.e. expected length of queue of system decreases.



	1	2	3	4	5
p	0.1	0.3	0.5	0.7	0.9
R	0.002111	0.006207	0.010133	0.013886	0.017469

Figure 4: Impact of reneging probability p on average reneging rate R of customers for γ= ξ=1.

Fig. 4 shows impact of reneging probability p on average reneging rate R of customers for γ= ξ=1. It has been observed from Fig. 4 that when we take λ=3 ,θ=0.2, μ=5, α_0=0.4, ξ=γ=1 ,as p increases a negligible increment have been observed in the value of average reneging rate R of customers.

6. Application of the Model

Here we are discussing real-life application of our model in a customer care service center:

Scenario

Consider a customer care service center of a software company which handles inquiries related to technical issues of customers. It is assumed that only one agent is hired to attend the call of customers. Then by using the concept of differentiated vacation the agent may handles essential services during busy period and the agent may handle basic queries or carry out non-essential services during working vacation. The agent may take shorter breaks during peak rush hour and may take longer vacation during quieter period. Vacation interruption may happen if there is an unanticipated spike in volume of calls or due to some serious technical issues. If any soft or intermittent failure occurs then the problem of call drop or delayed services arises which leads to customer dissatisfaction and customer may start reneging.

Model Application

This model can be used to analyze the performance of customer care service center by evaluating mean wait time, average queue length, retention rate of reneged customers. This can help in optimization of staff level to minimize waiting time and customers dissatisfaction. It also helps to analyze the impact of differentiated vacation and soft failure on customer which help the organization to identify the vulnerabilities in the system and implement contingency plan to enhance customer experience, service quality, customers retention, employees' satisfaction and mitigate risks. For retention of reneging customer, the company should implement a system which allows the organization to follow the information of reneged customers and address their issues so that customers remain loyal to the company.

7. Conclusion

In this Paper we have analyzed a M/M/1 queueing model with two different type of vacations in which first vacation which is longer is considered as working vacation with reneging of customers. The second vacation is of shorter duration in which interruption is considered. Various system probabilities, performance measures of the system like expected number of customers in the system E(L), sojourn time W, average reneging rate R, lost customers proportion P(L) have been calculated numerically and illustrated graphically by varying different system parameters. Finally, it has been concluded that this model can be used to analyze various performance measures of system to minimize wait time, queue length, lost customer proportion etc. which leads to improve customer retention, ultimately improving efficiency, productivity and customers satisfaction which helps to better understand and manage the dynamics of the system.

8. Future Scope:

In future one can extend this work by incorporating the concept of bulk arrival with threshold policy of service.

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